

Lund 1976

Break-Up Reactions Leading to

Resonant Final States.

by

Gunnar Ohlén and Tore Berggren[†]

Department of Mathematical Physics,

Lund Institute of Technology

S-220 07 Lund 7, Sweden.

Abstract: A theoretical model for knock-out reactions, which leave the residual nucleus in an unbound (resonant) state, has been worked out. The main problem is to describe the motion of a nucleon relative to an unstable core, which through an oscillating potential influences the motion of the nucleon even far away from the core. By a suitable redefinition of the potential, we have succeeded in eliminating some unphysical features of the interaction between the unstable core and the nucleon. In this model the origin of the imaginary part of the potential is well understood and some qualitative features are deduced. For instance it is shown that the imaginary part is positive and predominantly localized at the nuclear surface. This theory has been applied to the reaction ${}^6\text{Li}(p,2p){}^5\text{He}$. Even in a model where the target nucleus is described schematically our theory gives better fit to experimental data than other calculations.

[†] Work supported by the Swedish Atomic Research Council.

1. Introduction.

The binding of single nucleons in an atomic nucleus can be studied experimentally by means of break-up reactions, in which a nucleon of the kind considered is released from the nucleus. If the break-up is induced by a fast material particle, e.g. an electron or a proton, the reaction may be treated approximately as a direct knock-out process (see e.g. ref ¹⁾ and earlier papers reviewed there). It is well known that if a nucleon is knocked out from a deep inner shell, the residual nucleus is left in a highly excited state that is unstable against particle decay and thus has a finite lifetime. The decaying character of the final state has as a consequence that a substantial width must be assigned to this "hole" state. It has been noted by Herzcovitz et. al. ²⁾ that this width of the final states leads to some modifications of the form factor for the knocked-out nucleon. Also Shanta considered this problem ³⁾ and concluded in accordance with Herzcovitz et. al. that the form factor $\langle \tilde{\varphi}_f | \phi_i \rangle$ could be approximated as a solution to a Schrödinger-eq. with a non-Hermitian Hamilton operator.

In our reaction model we treat the final state as a complex energy eigenstate (Gamow or resonant state). This is possible since it has been shown by Berggren ⁴⁾ that these states together with the bound states form a biorthogonal set. In sect. 2 we use a method similar to that of Berggren ^{4,5)} to separate the resonant contribution from the background and thus derive an expression for the cross section for a direct knock-out reaction with a Gamow final state. We then discuss the high-energy limit and obtain the impulse approximation. Sect. 3 is then devoted to a discussion of the properties of the form factor. We define a form factor potential that has the properties which were assumed in refs. ^{2,3)}. As a test of our model we then apply it to the reaction ${}^6\text{Li}(p,2p){}^5\text{He}$ in sect. 4.

This is an ideal testing case as the unbound so-called ground state of ^5He is well approximated by a resonant state of the α -n system. Sect. 5 contains a discussion of the results obtained. Some formal matters have been collected in a few appendices.

2. The reaction model.

In this section we consider the reaction $A(p,2p)B$, where the nucleus B is left in a resonant state that decays by neutron emission: $B \rightarrow C + n$. We shall assume that C , henceforth called the core, is a stable nucleus. If the residual nucleus B were bound, we could assign a Møller operator

$$\Omega_{\alpha}^{\pm} = \lim_{t \rightarrow \mp \infty} e^{iHt/\hbar} e^{-iH_{\alpha}t/\hbar}$$

for every stable state α of B . But with a finite lifetime of the state α the time limit is meaningless. Instead we must take as a starting point the cross section for the three-body break-up reaction $A(p,2pn)C$ and try to separate the resonant and background contributions from each other.

When the reaction energy is above the three-body break-up threshold the main part of the cross section comes from two kinds of processes:

- i) The incoming proton interacts only with the knocked-out proton and the residual nucleus is left in a Gamow state.
- ii) The incoming proton interacts with both the proton and the neutron which leave the core simultaneously.

The first kind of process can in principle only happen for those neutron energies which are determined by the fact that the neutron wavefunction consists of purely outgoing waves at infinity. This is exactly the usual definition of a resonant state. The other kind of process should show no such energy structure and is considered to constitute the background. Our aim is now to separate the cross sections for these processes from each other.

To simplify the discussion we assume that the core C has no structure and has an infinite mass. (This last assumption is easily removed by introducing suitable Jacobian coordinates.) We further neglect the identity of the incoming and knocked-out proton. In order to calculate the cross

section for the reaction $A(p,2pn)C$ we take matrix elements of the T-operator ⁶⁾

$$T = T_{fi}^{(-)} = V_i + V_f \frac{1}{z - H} V_i \quad (2.1)$$

where V_i and V_f are the interactions that link the free fragments in the initial and final channels, respectively. The complex variable z stands for the total energy and will later approach the real axis as specified below. We further denote the incoming proton by p_1 and the knocked-out one by p_2 . The neutron is denoted by n and the core by C . As we are interested in the direct reaction in which p_1 knocks out p_2 , we neglect the interactions between p_1 and n and between p_1 and C . (These interactions could be retained and used to distort the wave-function for p_1 , but their removal is not crucial for the present discussion.)

The approximate full hamiltonain H can thus be written

$$H = T_{p_1} + T_{p_2} + T_n + V_{p_1 p_2} + V_{p_2 C} + V_{nC} + V_{p_2 n} \quad (2.2)$$

We simplify the notation by setting

$$V_{p_1 p_2} = V_{pp}$$

$$V_{p_2 C} = U_p$$

$$V_{nC} = U_n$$

$$V_{p_2 n} = V_{pn}$$

In this notation

$$V_i = V_{pp}$$

$$V_f = V_{pp} + U_p + U_n + V_{pn}$$

so that the T-operator can be written



Digitized by the Internet Archive
in 2026

https://archive.org/details/IA41553215_0004

$$T = V_{pp} + (V_{pp} + U_p + U_n + V_{pn}) \frac{1}{z-H} V_{pp} \quad (2.3)$$

We want to rewrite this operator so that the resonant structure of the final state is exhibited. These resonances are complex-energy eigenstates of the hamiltonian H_n defined by

$$H_n = T_n + U_n \quad (2.4)$$

Using the operator identity

$$\frac{1}{B} - \frac{1}{A} = \frac{1}{A} (A-B) \frac{1}{B} \quad (2.5)$$

with

$$A = z - H_f, \quad H_f = T_{p_1} + T_{p_2} + H_n$$

$$B = z - H$$

so that

$$A - B = V_{pp} + U_p + V_{pn}$$

we can rewrite $1/(z-H)$ as follows

$$\frac{1}{z-H} = \frac{1}{z-H_f} \{1 + (V_{pp} + U_p + V_{pn}) \frac{1}{z-H}\}$$

This expression can be used to rewrite eq. (2.3):

$$T = (1 + (V_{pp} + U_p + V_{pn}) \frac{1}{z-H} + U_n \frac{1}{z-H}) V_{pp} =$$

$$= (1 + U_n \frac{1}{z-H_f}) \{1 + (V_{pp} + U_p + V_{pn}) \frac{1}{z-H}\} V_{pp} \quad (2.6)$$

We now set $z = E + i\eta$ and let $\eta \rightarrow +0$. Then

$$1 + U_n \frac{1}{z-H_f} \rightarrow \Omega_n^{(-)\dagger} \quad (2.7)$$

where Ω denotes as usual the Møller wave operator. In this limit eq. (2.6) reads

$$T = \Omega_n^{(-)\dagger} T' \quad (2.8)$$

Here

$$T' = V_{pp} + (V_{pp} + U_p + V_{pn}) \frac{1}{z-H} V_{pp} \quad (2.9)$$

is the T-operator that is formally identical to the one for the reaction $A(p,2p)B$ with stable final states.

The relation between the T-matrix and the cross section is given by ⁶⁾

$$\frac{d^3\sigma}{d^3k_1 d^3k_2 d^3k_n} = \frac{(2\pi)^4 m}{k_0 \hbar^2} |\langle \underline{k}_1 \underline{k}_2 \underline{k}_n | T | \underline{k}_0 \phi_0 \rangle|^2 \delta(E - E_1 - E_2 - E_n) \quad (2.10)$$

where $\underline{k}_0$ is the wave-number for incoming proton and ϕ_0 is the ground state wave function for the target nucleus A. If we use the formula (2.8) the T-matrix elements can be expressed as

$$\langle \underline{k}_1 \underline{k}_2 \underline{k}_n | \Omega_n^{(-)\dagger} T' | \underline{k}_0 \phi_0 \rangle = \langle \underline{k}_1 \underline{k}_2 \phi_{\underline{k}_n}^{(-)} | T' | \underline{k}_0 \phi_0 \rangle \quad (2.11)$$

The wave-function $\phi_{\underline{k}_n}^{(-)}$ describes a continuum state for the neutron with incoming spherical waves as boundary condition. Before comparing with experiment expression (2.10) has to be integrated over all possible states of the undetected particle (the neutron). However, we can use the energy deltafunction to perform one integration immediately:

$$\frac{d^3\sigma}{d^3k_1 d\Omega_1 d\Omega_2} = \frac{(2\pi)^4 m k_1^2}{\hbar^2 k_0} \int d^3k_n \int k^2 dk_2 |\langle \underline{k}_1 \underline{k}_2 \phi_{\underline{k}_n}^{(-)} | T' | \underline{k}_0 \phi_0 \rangle| \delta(E - E_1 - E_2 - E_n). \quad (2.12)$$

Assuming $m_p = m_n = m$

$$E_1 + E_2 + E_n = \frac{\hbar^2}{2m} (k_1^2 + k_2^2 + k_n^2)$$

we can define

$$k_2(k_1, k_n) = \frac{1}{\hbar} (2m (E - \frac{\hbar^2}{2m} (k_1^2 + k_n^2)))^{1/2}$$

which gives

$$\frac{d^3 \sigma}{dk, d\Omega, d\Omega_2} = \frac{(2\pi)^4 m k_1^2 k_2(k_1, k_n)}{k^3 k_0} \int d\Omega_n \int_0^{k_n^{\max}} k_n^2 dk_n |\langle \underline{k}_1, \underline{k}_2(k_1, k_n) \phi_{\underline{k}_n}^{(-)} | T | \underline{k}_0 \phi_0 \rangle|^2. \quad (2.13)$$

The integrand in (2.13) is a real analytic function on the real axis, as can be seen by rewriting it as

$$M_{\underline{k}_1}(k_n) = \langle \underline{k}_1^*, \underline{k}_2(k_1^*, k_n^*) \phi_{\underline{k}_n^*}^{(-)} | T | \underline{k}_0 \phi_0 \rangle \langle \underline{k}_0 \phi_0 | T^\dagger | \underline{k}_1, \underline{k}_2(k_1, k_n) \phi_{\underline{k}_n}^{(-)} \rangle. \quad (2.14)$$

It now follows that M can be continued analytically into the complex k_n -plane. We shall assume (cf. ref ⁵⁾) that M is meromorphic in a symmetrical strip along the real k_n -axis. To explore the analytical structure of M we assume that U_n is a local potential of short range, so that the standard results of potential scattering theory can be applied ⁷⁾.

The function $\phi_{\underline{k}_n}^{(-)}(r)$ can be expanded as follows:

$$\phi_{\underline{k}_n}^{(-)}(r) = \sqrt{\frac{2}{\pi}} \frac{1}{k_n r} \sum_{\ell, m} i^\ell \psi_\ell(-k_n r) Y_{\ell, m}(\hat{r}) Y_{\ell, m}^*(\hat{k}) \quad (2.15)$$

where $\psi_\ell(-k_n r)$ can be written as ⁷⁾

$$\psi_\ell(-k_n r) = \frac{(-k_n)^{\ell+1} \phi_\ell(-k_n r)}{f_\ell(k_n)}. \quad (2.16)$$

In this formula $\phi_\ell(-k_n r)$ is an entire analytical function of k_n and $f_\ell(k_n)$ is the Jost function ⁷⁾. It is now clear that the only possible poles of (2.14) come from the zeros of the Jost function. Because of the two factors in (2.14) these poles appear as complex-conjugate pairs in the k_n -plane. From the relation

$$S_\ell(k) = \frac{f_\ell^*(k)}{f_\ell(k)} \quad (2.17)$$

it follows that these poles are the same as those used to define resonant states. In performing the k_n -integration from 0 to $k_n^{\max}$ we are now free to deform the integration path as long as no poles are swept over. We exploit this fact to split the integral into two parts. One part is the contribution from the pair of poles k_a and k_a^* that are associated with the resonant state having outgoing waves with momentum $\hbar k_a$. The other part is assumed to constitute the background. As integration path we choose a curve that describes a small semicircle above the pole k_a and another below the pole at k_a^* (see fig. 1). By letting the radii of these semicircles tend to zero we pick up a contribution that is $\pm i\pi$ times the residues at k_a and k_a^* . These considerations lead us to the following definition:

$$\left(\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dk_1} \right)_{R.C.} = C i\pi \left\{ \text{Res}_{k_n=k_a} (-M_{\underline{k}}(k_n)) + \text{Res}_{k_n=k_a^*} (M_{\underline{k}}(k_n)) \right\}. \quad (2.18)$$

(C is a factor easily deduced from (2.13).) The abbreviation R.C. as a subscript stands for "resonance contribution". To evaluate these residues we use the following formulae from ref. 8):

$$\text{Res}_{k=k_a} \left\{ k^2 \int d\Omega |\Phi_k^{(\pm)}\rangle \langle \Phi_k^{(\pm)}| \right\} = - \frac{1}{2\pi i} \sum_m |\tilde{\Phi}_{nm}\rangle \langle \Phi_{nm}|, \quad (2.19)$$

$$\text{Res}_{k=k_a^*} \left\{ k^2 \int d\Omega |\Phi_k^{(\pm)}\rangle \langle \Phi_k^{(\pm)}| \right\} = - \frac{1}{2\pi i} \sum_m |\Phi_{nm}\rangle \langle \tilde{\Phi}_{nm}|.$$

Here $|\Phi_{nm}\rangle$ is the resonant state with asymptotic wave number k_a and $|\tilde{\Phi}_{nm}\rangle$ is the time reverse of $|\Phi_{nm}\rangle$.

Using (2.18) and (2.19) to calculate (2.13) we get

$$\left(\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dk_1} \right)_{R.C.} = \frac{(2\pi)^4 m^2 k_1^2 k_2(k_1, k_n)}{\hbar^3 k_a} \text{Re} \left(\sum_m \langle \underline{k}, \underline{k}(k_1, k_n) | \tilde{\Phi}_{aem} | T | \underline{k}_0 \Phi_0 \rangle \cdot \langle \underline{k}_0 \Phi_0 | T^\dagger | \underline{k}_1 \underline{k}_2(k_1, k_n) \Phi_{aem} \rangle \right). \quad (2.20)$$

This expression should be compared with the cross section for the reaction $A(p,2p)B$ when B is left in a bound state

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dk_1} = \frac{(2\pi)^4 m^2 k_1^2 k_2(k_1 k_2)}{k^3 k_0} \sum_m |\langle \underline{k}_1 \underline{k}_2 \phi_{alm} | T' | \underline{k}_0 \phi_0 \rangle|^2. \quad (2.21)$$

It is evident that (2.20) reduces to (2.21) when the lifetime of ϕ_{alm} approaches infinity. This is an important consistency test of the theory.

In the limit of high-energy incoming protons the plane wave impulse approximation (PWIA) should be reasonably good for calculating the resonance contribution. To derive the PWIA of (2.20) we use the completeness relation

$$1 = \sum_{a'l'm'} \int |\underline{q} \Phi_{a'l'm'}\rangle d^3q \langle \underline{q} \tilde{\Phi}_{a'l'm'} | + \text{continuum contribution}. \quad (2.22)$$

In this expression the functions ϕ_{alm} are Gamow eigenstates of H_n . We then get

$$\begin{aligned} & \langle \underline{k}_1 \underline{k}_2 \tilde{\Phi}_{alm} | T' | \underline{k}_0 \phi_0 \rangle \\ &= \sum_{a'l'm'} \int \langle \underline{k}_1 \underline{k}_2 \tilde{\Phi}_{alm} | T' | \underline{k}_0 \underline{q} \Phi_{a'l'm'} \rangle d^3q \langle \underline{q} \tilde{\Phi}_{a'l'm'} | \phi_0 \rangle \\ &\approx \sum_{a'l'm'} \int \langle \underline{k}_1 \underline{k}_2 | T_{pp}^{free} | \underline{k}_0 \underline{q} \rangle \langle \tilde{\Phi}_{alm} | \Phi_{a'l'm'} \rangle d^3q \langle \underline{q} \tilde{\Phi}_{a'l'm'} | \Phi_0 \rangle. \end{aligned} \quad (2.23)$$

The sum is reduced to one term owing to the orthogonality relation for Gamow states ⁴⁾. These states are also orthogonal to the associated continuum states.

$$\langle \tilde{\Phi}_{alm} | \Phi_{a'l'm'} \rangle = \delta_{aa'} \delta_{ll'} \delta_{mm'} \quad (2.24)$$

To further simplify this expression we transform to the centre-of-mass system for the two protons. Some caution is needed to evaluate the scalar product $\langle \underline{q}_1^* | \underline{q}_2 \rangle$. We use the method of ref. ⁴⁾ to define

$$\langle \underline{q}_1^* | \underline{q}_2 \rangle = \lim_{\varepsilon \rightarrow 0} \frac{1}{(2\pi)^3} \int e^{-i\underline{q}_1 \cdot \underline{r}} e^{i\underline{q}_2 \cdot \underline{r}} e^{-\varepsilon r^2} d^3r \quad (2.25)$$

With this definition one easily gets

$$\langle \underline{q}_1^* | \underline{q}_2 \rangle = \delta(\underline{q}_1 - \underline{q}_2) \quad (2.26)$$

Collecting the results from this section we get

$$\left(\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} \right)_{R.C.} = R_c \left((PSF) \left(\frac{d\sigma}{d\Omega} \right)_{PP} \sum_m \left(\langle -\underline{k}^* \tilde{\Phi}_{alm} | \phi_0 \rangle \right)^2 \right) \quad (2.27)$$

where $\underline{k}$ is the recoil momentum of the residual nucleus (we now drop the assumption that C has infinite mass). PSF is the phase-space factor which in the symmetric coplanar case can be written

$$PSF = \frac{4k_1^2}{k^2 k_0} \cdot \frac{m^2 c^2 + k^2 k_1^2 \sin^2 \Theta}{(m^2 c^4 + k^2 c^2 k^2)^{1/2}} \quad (2.28)$$

Here 2Θ is the angle between the momentum vectors of the outgoing protons.

3. Properties of the form factor.

As was seen in the last section the nuclear structure information of the cross section is contained in the form factor or overlap integral

$$\langle -\underline{k} | \tilde{\varphi}_f | \Phi_i \rangle.$$

In this section capital Φ denotes eigenstates of the $C + p + n$ -system, while lower case φ denotes Gamow-states of the $C+n$ pair. The wave vector and the coordinate vector of the knocked-out proton are denoted by $\underline{k}$ and $\underline{r}$, respectively. It is more convenient to work with the Fourier transform which we denote by $\langle \underline{r} | \tilde{\varphi}_f | \Phi_i \rangle$. The form factor satisfies a system of integro-differential equations⁹⁾. As only bound final states were considered in the derivation in ref.⁹⁾, we will here repeat the arguments.

Let H_A be the Hamiltonian for the target nucleus A with ground state Φ_i . Then:

$$H_A | \Phi_i \rangle = E_i | \Phi_i \rangle \quad (3.1)$$

and also

$$H_n | \varphi_f \rangle = E_f | \varphi_f \rangle \quad (3.2)$$

Now H_A can be written

$$H_A = T_p + U_p + V_{pn} + H_n \quad (3.3)$$

where p refers to the knocked-out proton. If we multiply (3.1) with $\langle \underline{r} | \tilde{\varphi}_f |$ and use (3.3) we obtain

$$\langle \underline{r} | \tilde{\varphi}_f | T_p | \Phi_i \rangle + \langle \underline{r} | \tilde{\varphi}_f | U_p + V_{pn} | \Phi_i \rangle = (E_i - E_f) \langle \underline{r} | \tilde{\varphi}_f | \Phi_i \rangle \quad (3.4)$$

From ref.⁴⁾ we quote the completeness relation for resonant states.

$$1 = \sum_m \int | \underline{r}' | \varphi_m \rangle d^3 r' \langle \underline{r}' | \tilde{\varphi}_m | + \int \int_{L^+} | \underline{r}' | \varphi_{\underline{k}} \rangle d^3 r' d^3 k \langle \underline{r}' | \tilde{\varphi}_{\underline{k}} | \quad (3.5)$$

In this formula L^+ is a path that encloses the poles associated with the resonant states that are included in the sum. The second part in (3.5) will give rise to a resonance-continuum interference if used as intermediate states in (3.4). It is in the spirit of this work to deal exclusively with the specific resonant contribution, so this term will be dropped in the following discussion. Inserting (3.5) into (3.4) we get

$$-\frac{\hbar^2}{2m} \nabla_r^2 \langle \tilde{\varphi}_f | \Phi_i \rangle + \sum_m \int \langle \tilde{\varphi}_f | V_p + V_{pn} | \tilde{\varphi}_m \rangle d^3r' \langle \tilde{\varphi}_m | \Phi_i \rangle = (E_i - E_f) \langle \tilde{\varphi}_f | \Phi_i \rangle \quad (3.6)$$

To arrive at this equation we used the coordinate space representation of T_p , namely

$$T_p = -\frac{\hbar^2}{2m} \nabla_r^2.$$

The difference between eq. (3.6) and the one that applies to the case with a bound final state is twofold. First, the "eigen-value", $E_i - E_f$, has a nonzero imaginary part. Secondly, in the bras appear the states $\tilde{\varphi}_f$ which are part of a biorthogonal basis. This follows from the rule adopted by Berggren⁴⁾ that in forming scalar products the ordinary instead of the absolute square of the resonant functions is to be used. From this rule it follows that the interaction term will in this case also be non-real. It is also evident that the violent asymptotic behaviour of the Gamow states will affect the interaction term outside the nucleus.

The main interest of this section is to discuss the novel features of (3.6). However, eq. (3.6) is too complicated to solve directly, so we introduce two simplifying approximations. The first one is the assumption of zero-range nucleon-nucleon forces

$$V_{pn}(|\underline{r}-\underline{x}|) = -V_0 \delta(|\underline{r}-\underline{x}|) \quad (3.7)$$

Furthermore, we assume that only the diagonal term $m = f$ contributes to the sum in (3.6). These approximations do not affect the new aspects that interest us. The simplified version of eq. (3.6) reads:

$$\left(-\frac{\hbar^2}{2m} \nabla_r^2 + U_p(r) - \sum V_0 \tilde{\varphi}_f(r) \varphi_f(r)\right) \langle r \tilde{\varphi}_f | \Phi_i \rangle = (E_i - E_f) \langle r \tilde{\varphi}_f | \Phi_i \rangle \quad (3.8)$$

(The sum denotes summation over magnetic quantum numbers.)

Here the first potential term U_p describes the interaction between the core and the knocked-out proton and is unaffected by the finite life-time of the resonant state. Thus an ordinary Saxon-Wood type of potential would approximate it adequately if we neglect the Coulomb interaction. The other term in the form factor potential (FFP) will, however, deviate drastically from the bound state case. Writing

$$\langle r \tilde{\varphi}_f | \Phi_i \rangle = \frac{u(r)}{r} Y_l^m(\hat{r}) \quad (3.9)$$

we obtain

$$u''(r) + \{V(r) - \alpha^2\} u = 0 \quad (3.10)$$

with

$$\alpha^2 = 2m(E_i - E_f)/\hbar^2 ; \quad \text{Re } \alpha > 0;$$

$$V(r) = \frac{2m}{\hbar^2} (U_p(r) - V_0 \sum \tilde{\varphi}_f(r) \varphi_f(r) + \text{contr. part})$$

Now the resonant state has an asymptotic behaviour as follows

$$\varphi_f(r) \sim Y_l^m(\hat{r}) \frac{e^{ikr}}{r}, \quad r \rightarrow \infty \quad (3.11)$$

$$k = \kappa - i\gamma ; \quad \kappa, \gamma > 0.$$

This gives for the density

$$\sum \tilde{\varphi}_+(r) \varphi_+(r) \sim \frac{e^{2ikr}}{r^2}, \quad r \rightarrow \infty, \quad (3.12)$$

where we have used the definition ⁴⁾

$$\tilde{\varphi}_+(r) = \frac{1}{r} \tilde{u}_{\ell m}(r) Y_{\ell}^m(\hat{r}). \quad (3.13)$$

Inside the nucleus the Gamow wave function is almost real as found in refs. ^{10,11)}. This fact is also confirmed by our numerical calculations, at least for longlived states with $\ell > 0$. (For s-states no definite conclusions can be drawn.) The real part of the resonant wave function resembles very much that of a bound state inside the nucleus. At the surface, however, the imaginary part grows and is comparable in magnitude to the real part outside the nucleus. These properties show that the real part of the FFP should be similar to an ordinary bound state potential inside the nucleus. On the other hand, the imaginary part of the FFP is small inside the nucleus, but grows towards the nuclear surface region. There it is negative as has already been found in ref. ¹²⁾.

Having established the properties of the FFP inside and on the surface of the nucleus we proceed to discuss its asymptotic behaviour. As can be seen from (3.12) and (3.10) it behaves as

$$V(r) \sim -V_0 \frac{e^{2ikr}}{r^2} \quad (3.14)$$

when r is large. This asymptotic behaviour differs dramatically from what is expected when the residual nucleus is left in a stable eigenstate. However, we will prove that the form factor itself behaves essentially as that of a bound final state far away from the nucleus providing that a slight redefinition is made of it. From (3.14) we see that the FFP oscillates with increasing amplitude for large r . This property gives rise to rapid oscillations with decreasing wavelength in the form-factor

itself (cf. Appendix A). This means that very high momentum components are present in the form factor, but they are associated with processes taking place far away from the nucleus. In order to eliminate these rapid oscillations that are of no interest in a reaction with limited momentum transfer, we define a smoothed form factor as

$$V_u(r) = \int_0^{\infty} f(r-r') u(r') dr'. \quad (3.15)$$

Here f is a normalized regularization function that will be specified in appendix A. By partial integration we obtain without difficulty

$$V_u' = V_u,$$

and

$$V_u'' = -f(r)u'(0) + \int_0^{\infty} f(r-r') u''(r') dr'. \quad (3.16)$$

If we multiply eq. (3.10) with $f(r-r')$ and integrate we get

$$V_u'' - \alpha^2 V_u - f(r)u'(0) + \int_0^{\infty} f(r-r') V(r') u(r') dr' = 0. \quad (3.17)$$

It is shown in appendix A that u can be written

$$u(r) = e^{-\alpha r} g(r), \quad (3.18)$$

where g is an entire analytic function. We further assume that f decreases rapidly so that the term $f(r)u'(0)$ can be dropped in studying the asymptotic behaviour of eq. (3.17). Using (3.18) the interaction term in eq. (3.17) can be rewritten

$$\int_0^{\infty} f(r-r') V(r') u(r') dr' = e^{-\alpha r} \int_0^{\infty} e^{\alpha(r-r')} f(r-r') V(r') g(r') dr'. \quad (3.19)$$

If we now assume that

$$V_u(r) \sim e^{-\alpha r}, \quad r \rightarrow \infty \quad (3.20)$$

(subject to later confirmation) eq. (3.17) can in the asymptotic region be written

$$V_u'' - \alpha^2 V_u + V_u W(r) = 0, \quad r \rightarrow \infty. \quad (3.21)$$

We denote

$$W(r) = \int_0^\infty e^{\alpha(r-r')} f(r-r') V(r') g(r') dr' \quad (3.22)$$

as the effective form factor potential (EFP). With this redefinition of the form factor we have effectively smoothed out the unphysical oscillations and both the EFP and the form factor itself behave asymptotically correct. The proof of these statements can be found in appendix A. In appendix B we give another method to redefine the FFP which takes account to the fact that the resonant state was created a finite time ago. To summarize the discussion we note the following important properties of the EFP:

- a It is real and approximately proportional to the nuclear density inside the nucleus.
- b At the nuclear surface it has a positive (creative) imaginary part.
- c The asymptotic behaviour is such that the interaction is of finite range.

We note in passing that the term optical potential that appears in the literature is in this connection ^{2,3)} hardly appropriate. The name optical potential should be reserved for theories, where an absorptive potential is needed to simulate inelastic processes that are not treated explicitly. From the derivation of the EFP it is clear that this is not the case here. Another difference is that we prefer to view the formfactor $\langle \tilde{\varphi}_f | \phi_i \rangle$ as the wave function for the knocked-out nucleon at the moment the reaction occurs. It seems to us that the name "hole states" can be used only for the states φ_f . (This was already pointed out in ref. ¹²⁾.)

4. The reaction ${}^6\text{Li}(p,2p){}^5\text{He}$.

As a testing case for the model developed in the previous sections we have made calculations on the reaction ${}^6\text{Li}(p,2p){}^5\text{He}$. It is well known that the nucleus ${}^5\text{He}$ is unstable and decays into an α -particle and a neutron with a lifetime of 2×10^{-21} seconds. This means that the "ground state" of ${}^5\text{He}$ is a resonant one. Furthermore the structure of ${}^5\text{He}$ is well approximated as an inert α -core with a neutron outside. These facts make this reaction ideal for an application of our model. This reaction has been studied experimentally ¹³⁾. It was then found that the angular distribution for the ground state transitions shows peculiar features that deviate from other p-shell nuclei. The shape of the distribution indicates a mixture of p- and s-states.

The first stage in our calculations was to investigate the low-lying resonances of the n- α system. Nussenzweig ¹⁴⁾ has calculated resonant energies in a square-well potential. His results are in agreement with general theoretical predictions ¹⁵⁾. The most remarkable fact is that there is a substantial difference between s-states and states with higher angular momentum. This is easy to understand because a neutron in an s-state has no barrier to penetrate so the only circumstance that can prevent it from escaping is the wave-mechanical reflexion at the surface. When the ℓ -value is greater than zero, however, the centrifugal potential gives rise to a barrier making the resonances narrower. In fig. 3 we have depicted the threshold behaviour of the S-matrix poles as the strength of the interaction is weakened and transforms the bound states into resonant ones. For the higher partial waves ($\ell > 0$) the transition from bound to Gamow states occurs directly and the pole trajectory stays close to the real axis. In contrast to this the s-state first goes over into an anti-bound state, and only when the interaction has weakened substantially does

it appear as a true resonant state ($|\operatorname{Re} k_n| > |\operatorname{Im} k_n|$).

When performing our calculation we choose a Saxon-Wood potential for the n - α interaction.

$$V(r) = V_c \left(f(r) + v \left(\frac{\hbar}{2mc} \right)^2 \frac{1}{r} \cdot \frac{df}{dr} \cdot \frac{\vec{L} \cdot \vec{S}}{2 \hbar^2} \right),$$

$$f(r) = \left(1 + \exp \left((r - R_N/a_N) \right) \right)^{-1}. \quad (4.1)$$

With the help of this potential resonant states were calculated numerically as described in appendix C. The parameters of (4.1) were adjusted so as to give agreement with the experimental energy and width of the $1p_{3/2}$ ground state¹⁶⁾. During the fitting procedure the radius R_N , the depth V_c and the spin-orbit coupling constant v were held constant and we only varied the diffuseness parameter a_N . The values of the parameters used can be found in table 1. As a test of the n - α interaction obtained in this way we calculated the s-wave phase shift. It was found to agree well with the experimental values¹⁷⁾ apart from an additional constant π (180°) corresponding to the $1s$ bound state in accordance with Levinson's theorem⁷⁾. Having determined the interaction we calculated several resonant states with low energy. The results of these calculations can be found in table 2. In fig. 4 we have indicated the positions of the corresponding poles in the complex k_n -plane. It is evident that the results are in agreement with the theory as discussed earlier and shown in fig. 3. It is worth noting that the $2s_{1/2}$ state has a lower energy than the $1d_{5/2}$ state. In the usual shell-model these levels occur in the opposite order. In order to investigate the influence of the $2s_{1/2}$ resonant pole on the cross section we must estimate what fraction of the time that the knocked-out proton spends in this orbit. Since we believe that the filling in of the minima in the angular distribution is due to the s-state pole the following ansatz for the 1^+ ground state wave function of ${}^6\text{Li}$ is the simplest one

that can explain the experimental data:

$$|\psi\rangle = A (1s)^4 (1p_{3/2})^2 + B (1s)^4 (2s_{1/2})^2. \quad (4.2)$$

In this formula $(1s)^4$ denotes the α -core. The amplitudes A and B were calculated from a simple diagonalization. To calculate the diagonal elements we used a δ -force with a strength $V_0 = -300$ MeV. The non-diagonal elements were used as free parameters adjusted so as to give the right ground state energy of ${}^6\text{Li}$. Using this simple Hamiltonian we obtained the following values for the normalized amplitudes:

$$A = 0.98 + 0.05i$$

$$B = 0.17 - 0.15i$$

(These values are very insensitive to the strength of the interaction.)

In order to obtain the cross sections we calculated the form factors for the corresponding final states from eq. (3.8) using the EFP. In principle it would be possible to use (3.22) to calculate the EFP. This turned out to be difficult from a computational point of view so we used an ansatz based on the general properties of the EFP which were obtained in sect. 3. For the real part an ordinary Saxon-Wood potential was used while the imaginary part had the form

$$\text{Im } W(r) = C \cdot r \cdot \exp(-((r - R_0)/a_0)^2). \quad (4.2)$$

This ansatz was used for both the $1p_{3/2}$ and $2s_{1/2}$ final states although the arguments of sect. 3 is doubtful for very short-lived final states such as the $2s_{1/2}$ state. The parameters of the EFP was varied so as to give the right "eigen-value"

$$E_i - E_f = - \frac{\hbar^2}{2\mu} \alpha^2.$$

The result of this fit can be found in table 3. In figs. 6a and b the

different EFP:s are plotted. Note the large radius for the s-case. The corresponding cross sections are presented in figs. 8a and b. As the imaginary part in the $\ell = 0$ case is of the same magnitude as the real part there is a large resonance-background interference. This is natural as the life-time of this resonant state is very short, about the same as the reaction time. It is only in the cases when the lifetime is much longer than the reaction time that one can neatly separate the background and resonant contributions from each other. In spite of this and the very schematic treatment of the target nucleus the agreement with experimental data is rather good (see fig. 9a). We believe that this calculation shows that the filling in of the typical p-shell minimum in the angular distribution is due to the presence of the low-lying $2s_{1/2}$ resonant pole. In a more realistic calculation it would be necessary to include the background effect which is only possible in a Faddeev type treatment of the T-matrix.

5. Conclusions and discussion.

The main theoretical result of this work is that it is possible to define a Schrödinger equation for the form factor even in the case of resonant final states. The interaction term (EFP) in this equation has some important properties which follow naturally in our treatment. The non-hermiticity of the EFP that was assumed in refs. 2,3) is directly and dynamically related to the decay of the final Gamow state. In our numerical application to the reaction ${}^6\text{Li}(p,2p){}^5\text{He}$ we focus attention to the low-lying $2s_{1/2}$ Gamow state. This state is, admittedly, too short-lived to be used with any claim of realism as a final state in the model developed in sects. 2 and 4. Still, in order to estimate the effect of this low-lying pole we have applied our formalism to it as a final state. Despite the questionable realism of this procedure it is remarkable that the inclusion of this state in the ${}^6\text{Li}$ ground state gives almost the right filling in of the minimum in the angular distribution.

A natural application of our model would be to the knocking out of inner shell nucleons resulting in deep "hole"-state. The "hole"-state produced by removing rapidly an inner shell nucleon is a highly excited state lying above the particle emission threshold. In this initial stage the excess energy is shared by many nucleons in the residual nucleus. Before a nucleon (or cluster of nucleons) can escape it must acquire a sufficiently large amount of this energy. In the shell-model such a redistribution of energy can take place as a series of two-body collisions, resulting in 2-hole-1-particle, 3-hole-2-particle states and so on, until the "exit doorway state" is reached. There exist several such states, each of them being a resonant eigenstate of the Hamiltonian for the residual nucleus. These states form a group with an over-all width ("spreading width")¹⁹⁾, determined roughly by the time scale of the process just de-

scribed. It is this width that is seen in a low-resolution experiment. The individual widths of the states of the group are smaller and these resonant states are those which should be used as final states in our model. It is however, clear that the concept "spreading width" can be defined only with respect to a given model, such as the single particle model.

Appendix A.

Asymptotic behaviour of the formfactor.

In section 3 we derived an equation for the formfactor:

$$-u'' + (\alpha^2 - V(r))u = 0. \quad (A.1)$$

The potential V has the asymptotic behaviour

$$V(r) \sim \frac{\lambda}{r^2} e^{2ikr}. \quad (A.2)$$

To get an analytically solvable equation we disregard the r^{-2} dependence of V which affects the asymptotic form very little. For large r we then have:

$$-u'' + (\alpha^2 - \lambda e^{2ikr})u = 0. \quad (A.3)$$

The solution can be expressed in terms of the Bessel function $J_\nu(x)$ as

$$u(r) = C J_{\frac{i\alpha}{k}} \left(\frac{i\lambda}{k} e^{ikr} \right). \quad (A.4)$$

There is actually also a solution

$$J_{-\frac{i\alpha}{k}} \left(\frac{i\lambda}{k} e^{ikr} \right)$$

which we disregard. The reason for this will be clear later. From ref. 18) we get

$$J_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{n=0}^{\infty} \frac{\left(-\frac{z^2}{2}\right)^n}{n! \Gamma(\nu+n+1)}, \quad (A.5)$$

where the series

$$\sum_{n=0}^{\infty} \frac{\left(-\frac{z^2}{2}\right)^n}{n! \Gamma(\nu+n+1)} \quad (A.6)$$

is an entire analytic function of z . This gives

$$u(r) = B e^{-\alpha r} \sum_{n=0}^{\infty} a_n e^{2iknr} \quad (\text{A.7})$$

Here

$$B = 4 \left(\frac{i\lambda}{k} \right)^{\frac{i\alpha}{k}}$$

and

$$a_n = \frac{\left(-\frac{i\lambda}{2k} \right)^n}{n! \Gamma\left(\frac{i\alpha}{k} + n + 1\right)}.$$

Introducing a regularisation function

$$f(s) = \frac{1}{\sqrt{\pi} \beta} e^{-s^2/\beta^2}, \quad (\text{A.8})$$

we obtain for the "smoothed" formfactor

$$V(r) = \frac{B}{\sqrt{\pi} \beta} \int_0^{\infty} e^{-\alpha r'} \sum_{n=0}^{\infty} a_n e^{2iknr'} e^{-(r-r')^2/\beta^2} dr'. \quad (\text{A.9})$$

It is legitimate due to uniform convergence to interchange the order between integration and summation in (A.9)

$$V(r) = \frac{B}{\sqrt{\pi}} \sum_{n=0}^{\infty} a_n e^{-r^2/\beta^2} e^{w^2} \int_{-w}^{\infty} e^{-s^2} ds, \quad (\text{A.10})$$

where

$$w = \frac{1}{\beta} (ikn\beta^2 + r - \alpha\beta^2/2).$$

In order to examine the asymptotic behaviour of v we use the well-known asymptotic expansion

$$e^{w^2} \int_{-w}^{\infty} e^{-s^2} ds \sim \sqrt{\pi} e^{w^2} \Theta(\text{Re } w) - \frac{1}{2w} \left(1 - \frac{1}{2w^2} + \dots \right). \quad (\text{A.11})$$

θ denotes the unit step function defined as:

$$\theta(s) = \begin{cases} 0, & s < 0, \\ 1, & s > 0. \end{cases}$$

Now for fixed β and large r , $\text{Re}(w)$ is greater than zero independently of n , so keeping only the leading term in (A.11) v can be written

$$V(r) \sim e^{-\alpha r} e^{\alpha^2 \beta^2 / 4} B \sum_{n=0}^{\infty} a_n \exp(-k^2 n^2 \beta^2 + 2ikn(r - \beta^2 \alpha / 2)), \quad r \rightarrow \infty. \quad (\text{A.12})$$

Note that for $\beta \rightarrow 0$, we recover the original series, corresponding to the fact that in this limit the regularisation function becomes a δ -function.

For

$$n > \frac{2\gamma r}{(\mathcal{X}^2 - \gamma^2)\beta^2}, \quad k = \mathcal{X} - i\gamma \quad (\text{A.13})$$

the function

$$\exp(-k^2 n^2 \beta^2 + 2iknr) \quad (\text{A.14})$$

is less than unity in absolute value so if we split the sum in (A.12) into two parts,

$$\sum_{n=0}^{\infty} = \sum_{n=0}^{n_1} + \sum_{n_1+1}^{\infty}, \quad (\text{A.15})$$

where we choose

$$n_1 = \text{int} \left(2\gamma r / ((\mathcal{X}^2 - \gamma^2)\beta^2) \right),$$

the last part is negligible. The first part is, if $\mathcal{X} > \gamma$, dominated by the first term, so we obtain as a crude but conservative estimate

$$V(r) \sim e^{-\alpha r} e^{\alpha^2 \beta^2 / 4} B a_0 \frac{2\gamma r}{(\mathcal{X}^2 - \gamma^2)\beta^2}, \quad r \rightarrow \infty, \quad (\text{A.16})$$

the last factor in the right member approximating the number of non-

negligible terms. The smoothed formfactor thus has a decent behaviour far away from the nucleus. If we choose an EFP that is of finite range the solution to eq. (3.21) behaves asymptotically as a bound state.

Appendix B.

Alternative treatment of the formfactor.

In this appendix we discuss an alternative method to deduce the asymptotic behaviour of the form factor. As the Gamow wave-functions describe exponentially decaying states it is clear that they were created a finite time ago. This makes the transition from a proper time dependent description of the reaction process to a time independent one troublesome. The convergence factor $\exp(-\epsilon r^2)$ used in ref. ²⁾ to define scalar products between resonant states is a reflection of this fact, as it cuts out the part of the wave-function that corresponds to "old" states. When the limit $\epsilon \rightarrow 0$ is taken the result obtained does not depend on the lifetime of the states and a proper time independent theory can be constructed. The resonant final states considered in this paper are created during the reaction, and the formfactor describes the motion of the knocked-out nucleon relative to the unstable residual nucleus. In the same spirit as in ref. ²⁾ we introduce a convergence factor into the FFP defined by (3.6) and get

$$V_{\Sigma}(r) \sim \lambda \frac{e^{-\epsilon r^2} \cdot e^{2ikr}}{r^2} \quad (B.1)$$

outside the core. The convergence factor $\exp(-\epsilon r^2)$ can be interpreted as to cut out that part of the Gamow-state which corresponds to decays prior to the collision. It is therefore worthwhile to examine the asymptotic behaviour of the solutions to

$$u_{\Sigma}'' - \alpha^2 u_{\Sigma} + V_{\Sigma}(r) u_{\Sigma} = 0. \quad (B.2)$$

As

$$V_{\Sigma}(r) = O\left(\frac{1}{r^3}\right), \quad r \rightarrow \infty,$$

it is wellknown that

$$V_{\varepsilon}(r) \rightarrow e^{-\alpha r}, \quad r \rightarrow \infty.$$

However, to be able to deduce the behaviour of v as $\varepsilon \rightarrow 0$ for large r we need better estimates. In order to do this we rewrite (B.2) as

$$u_{\varepsilon}'' - \alpha^2 u_{\varepsilon} = -\lambda e^{2ikr} e^{-\varepsilon r^2} u_{\varepsilon}. \quad (B.3)$$

(We disregard the unimportant r^{-2} factor).

Now as $u_{\varepsilon} \sim e^{-\alpha r}$, when $r \rightarrow \infty$, although the two terms on the left hand side will each be larger than the right hand side, they almost cancel. It is therefore possible to treat the right hand side as a perturbation.

The equation

$$f'' - \alpha f = g$$

has a particular solution

$$f(r) = e^{-\alpha r} \int_r^{\infty} e^{2\alpha t} \left(\int_t^{\infty} g(s) e^{-\alpha s} ds \right) dt. \quad (B.4)$$

With

$$g(r) = -\lambda e^{2ikr - \varepsilon r^2} u_{\varepsilon} \approx \lambda e^{2ikr - \varepsilon r^2 - \alpha r}, \quad (B.5)$$

we get after some algebra

$$u_{\varepsilon}(r) \sim e^{-\alpha r} \left(-\lambda \frac{e^{-k^2/\varepsilon - \varepsilon r^2}}{2\varepsilon^2} + H \right), \quad r \rightarrow \infty. \quad (B.6)$$

In deriving (B.6) we have used the asymptotic formula (A.10) and have added the solution to the homogenous equation, viz.

$$Ae^{-\alpha r}.$$

If we now let $\epsilon \rightarrow 0$ in (B.6) we can see that the asymptotic property

$$u_{\epsilon}(r) \sim A e^{-\alpha r}$$

is valid even in the limit $\epsilon \rightarrow 0$, provided $\text{Re } k^2 > 0$, i.e. $\kappa^2 > \gamma^2$.

From this discussion it is evident that the formfactor

$$u(r) = \lim_{\epsilon \rightarrow 0} u_{\epsilon}(r) \tag{B.7}$$

satisfies a Schrödinger equation with a potential that is of finite range.

Appendix C.

Remarks concerning the numerical treatment.

In this appendix we will give a short description of the numerical calculation of resonant states. These states are complex energy solutions to the radial Schrödinger equation:

$$-u_e'' + \left(\frac{2m}{\hbar^2} V(r) + \frac{l(l+1)}{r^2} \right) u_e = k^2 u_e, \quad (C.1)$$

with boundary conditions,

$$\begin{aligned} u_e(0) &= 0 \\ u_e(r) &\rightarrow e^{ikr}, \quad r \rightarrow \infty, \quad k = \mathcal{E} - i\gamma. \end{aligned} \quad (C.2)$$

With a suitable starting value of k eq. (C.1) is integrated numerically using standard routines outwards from the origin and inwards from the asymptotic region, taking account of the appropriate boundary conditions. The two solutions (denoted by u_{e1} and u_{e0} respectively) are then matched at a suitable radius R_m . The condition to be met is that the logarithmic derivatives are equal for $r = R_m$. The eigenvalue k^2 thus appears as a zero of the function:

$$F(k) = \frac{u_{e0}'(R_m)}{u_{e0}(R_m)} - \frac{u_{e1}'(R_m)}{u_{e1}(R_m)} \quad (C.3)$$

An iterative procedure is needed to solve this two-dimensional system of non-linear equations. In the calculations reported in ref. ¹²⁾ we used a method of Newton - Raphson type as used for bound state calculations (see ref. ²⁰⁾) but this method was found inadequate to handle very short-lived states.

We have used for this purpose the routine NSOIA from the Harwell Subroutine Library, which combines the Steepest Descent and Newton - Raphson methods. This routine proved to be very reliable and effective also in the rather extreme cases encountered here.

References

1. G. Jacob and Th. A. J. Maris, Rev.Mod.Phys. 38 (1966) 121; 45 (1973) 6
2. V.E. Herscovitz, G. Jacob and Th.A.J. Maris, Nucl.Phys. A109 (1968) 478;
V.E. Herscovitz, Nucl.Phys. A161 (1971) 321
3. R. Shanta, Nucl.Phys. A199 (1973) 624
4. T. Berggren, Nucl.Phys. A109 (1968) 265
5. T. Berggren, Nucl.Phys. A169 (1971) 353
6. R. Newton, Scattering Theory of Waves and Particles, (Mc Graw-Hill, New York, 1966) Chapter 8.
7. R. Newton, J.Math.Phys. 1 (1960) 319
8. T. Berggren, Phys.Lett. 44B (1973) 23
9. T. Berggren, Nucl.Phys. 72 (1965) 337
10. G. Gyarmati, R.G. Lovas and J. Zimányi, Phys.Lett. 35B (1971) 549
11. T. Berggren and B. Gyarmati, to be published
12. T. Berggren and G. Ohlén, Lett. el Nuov.Cim. 7 (1973) 701
13. G. Tibell, O. Sundberg and P.U. Renberg, Ark.Fys. 25 (1963) 433;
R.K. Bhowmik, C.C. Chang, P.G. Roos and H.D. Holmgren, Nucl.Phys. A226
(1974) 365
14. H.M. Nussenzweig, Nucl.Phys. 11 (1959) 499
15. Taylor, Scattering Theory, (John Wiley & Sons, New York 1972), Chapter 13
16. T. Lauritsen and F. Ajzenberg-Selove, Nucl.Phys. 78 (1966) 1
17. F. Demains, G. Pisent, G. Poiani and C. Villi, Phys.Rev. 125 (1961) 318
18. Jahnke, Ende and Lösch, Tables of Higher Functions (Mc Graw-Hill, New York 1960)
19. C.A. Engelbrecht and H.A. Weidenmüller, Nucl.Phys. A184 (1972) 385
20. G.E. Brown, J.H. Gunn and P. Gould, Nucl.Phys. 46 (1963) 598

Table Captions.

Table 1. Potential parameters for the neutron-alpha interaction.

Table 2. Single-particle levels in ${}^5\text{He}$ calculated using a Saxon-Woods potential with the parameters of Table 1.

Table 3. Parameters for the effective form factor potential, eq. (4.2)

Table 1.

$$V_c = -50 \text{ MeV}$$

$$V = 20$$

$$R_N = 2.05 \text{ fm}$$

$$a_N = 0.35 \text{ fm}$$

Table 2.

Level	Energy (MeV)	Wavenumber ((Fm) ⁻¹)
1s _{1/2}	-16.06	0.785i
1p _{3/2}	0.80-0.25i	0.177-0.027i
1p _{1/2}	5.22-8.11i	0.534-0.292i
2s _{1/2}	6.58-46.93i	1.018-0.885i
1d _{5/2}	16.10-11.04i	0.827-0.256i

Table 3.

	1p _{3/2}	2s _{1/2}	
V _c	-52	-50	MeV
V	20	20	
R _N	2.33	4.0	fm
a _N	0.35	1.0	fm
C	0.55	31.0	MeV/fm
R ₀	4.5	2.5	fm
a ₀	1.0	1.0	fm

Figure captions.

1. Integration paths in the complex k_n -plane used to define the resonant contribution to the cross section.
2. Typical shape of the cross section as a function of energy. The resonant contribution is obtained by integrating over the shaded area.
3. S-pole trajectories in the complex k -plane.
4. Resonant poles for the α -n system.
5. Neutron resonant wave functions in ${}^5\text{He}$.
 - a. $1p_{3/2}$
 - b. $2s_{1/2}$
6. Effective form factor potentials
 - a. For $1p_{3/2}$ protons
 - b. " $2s_{1/2}$ "
7. Form factors
 - a. $1p_{3/2}$
 - b. $2s_{1/2}$
8. Cross sections
 - a. $1p_{3/2}$ final state
 - b. $2s_{1/2}$ " "
9. Total cross section
 - a. Real part
 - b. Imaginary part

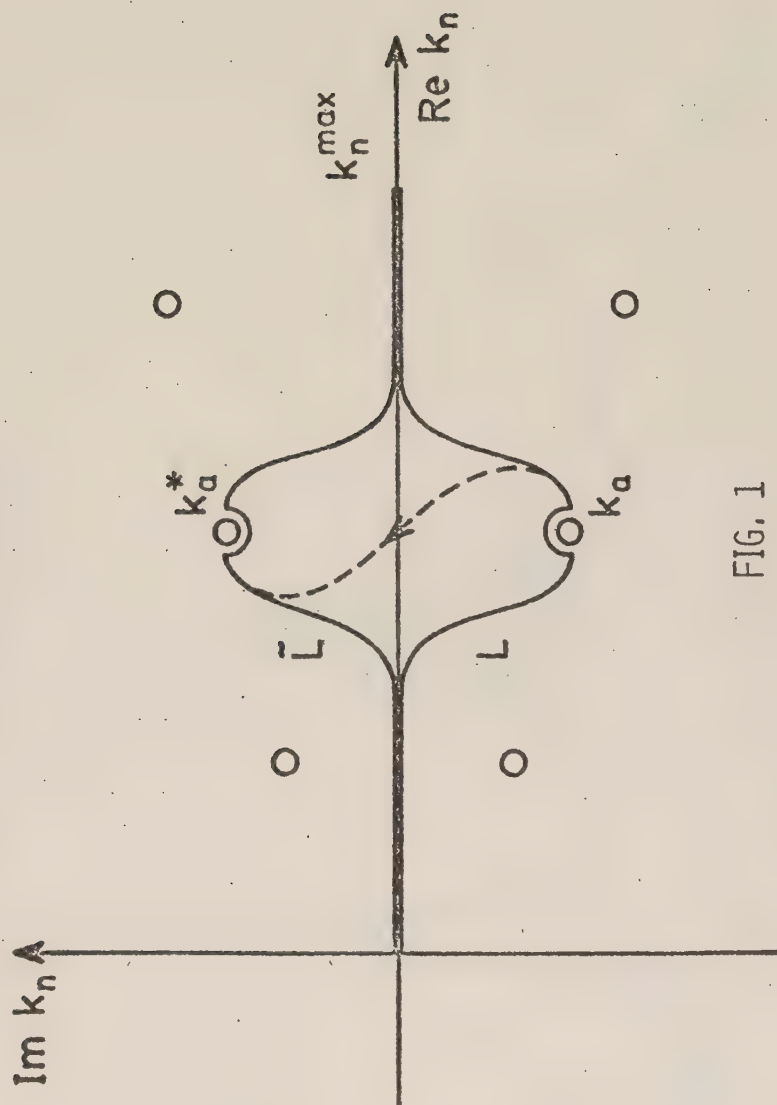


FIG. 1

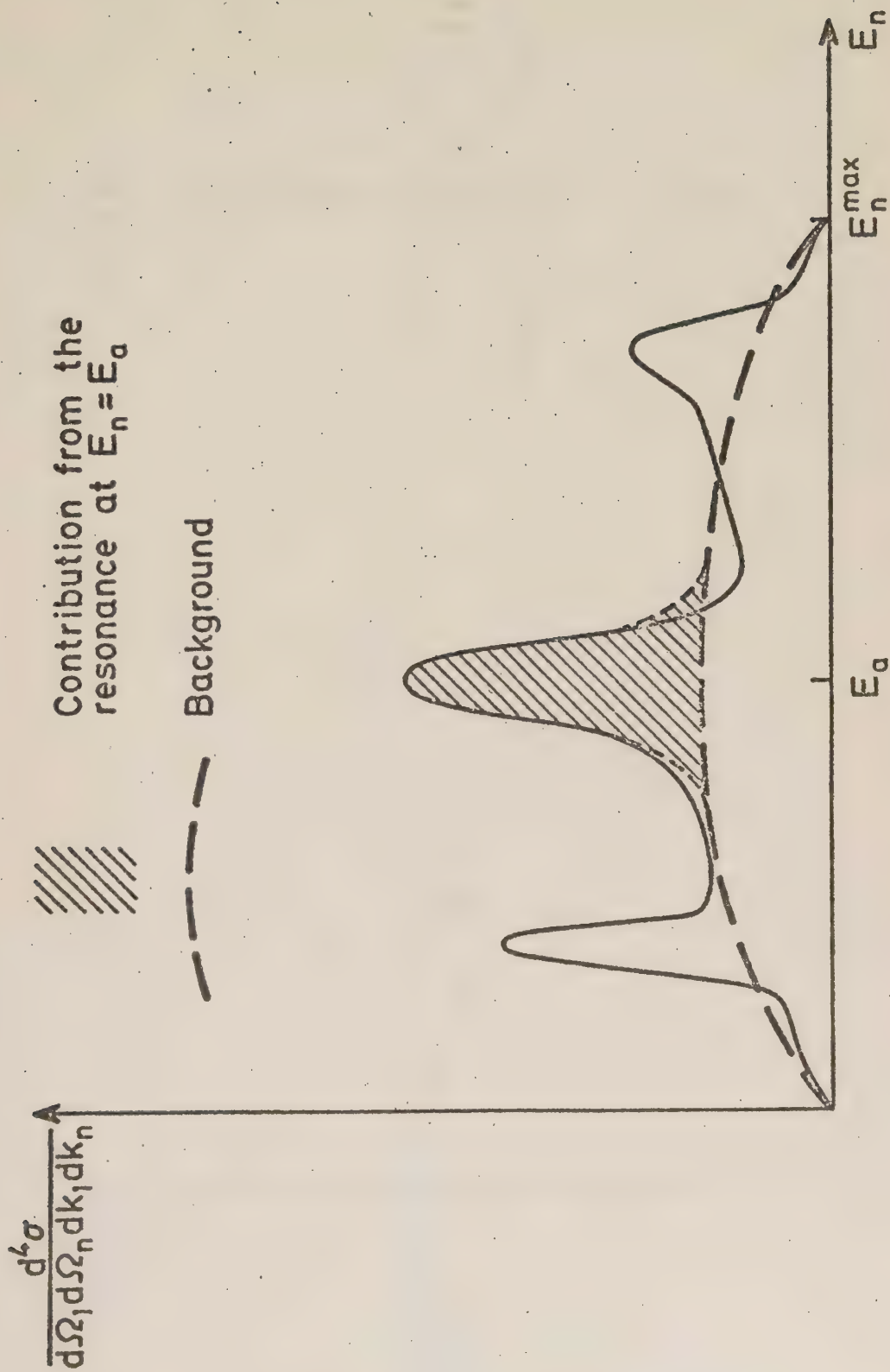


FIG. 2

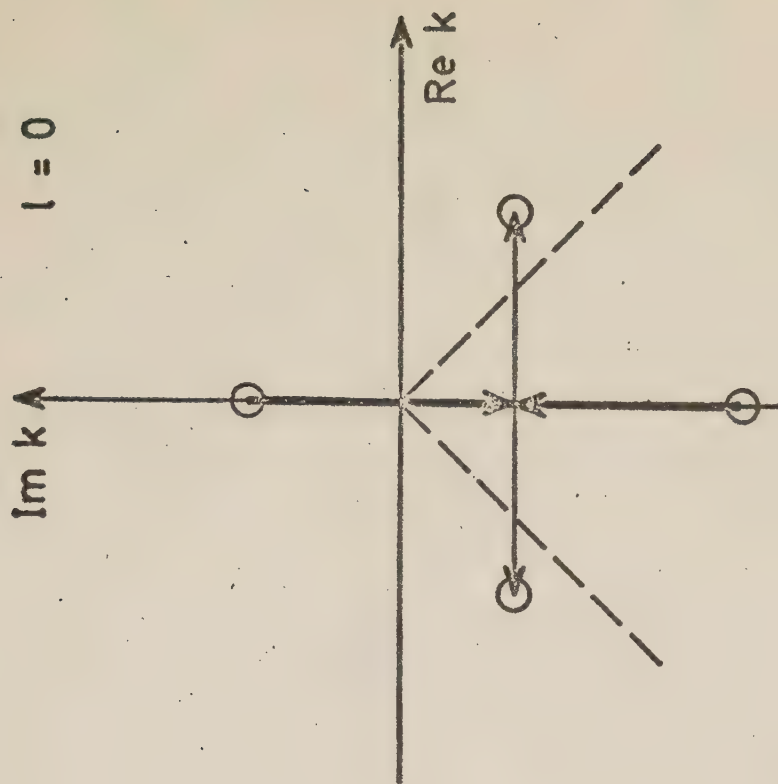
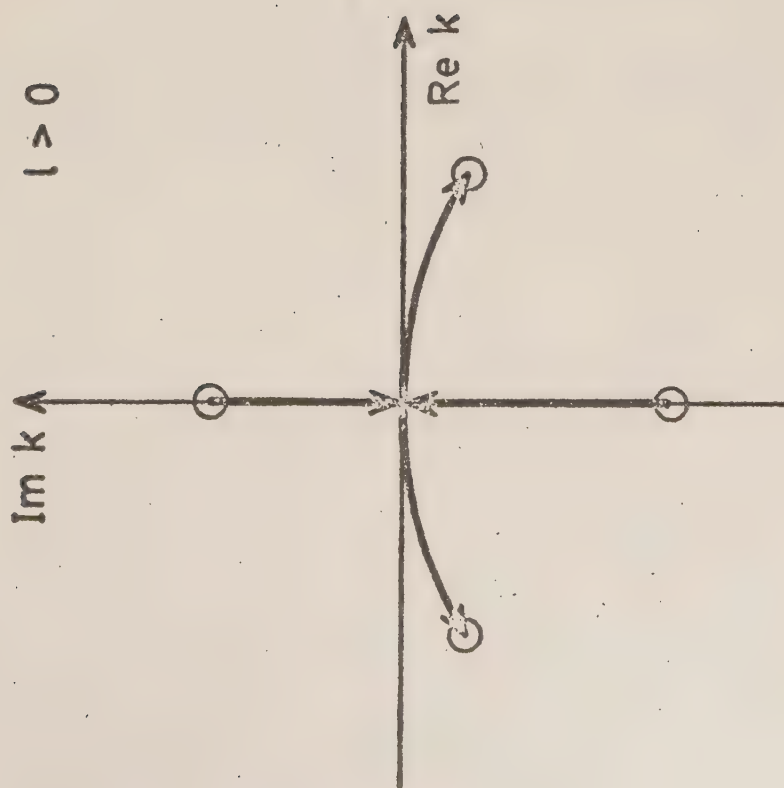


FIG. 3

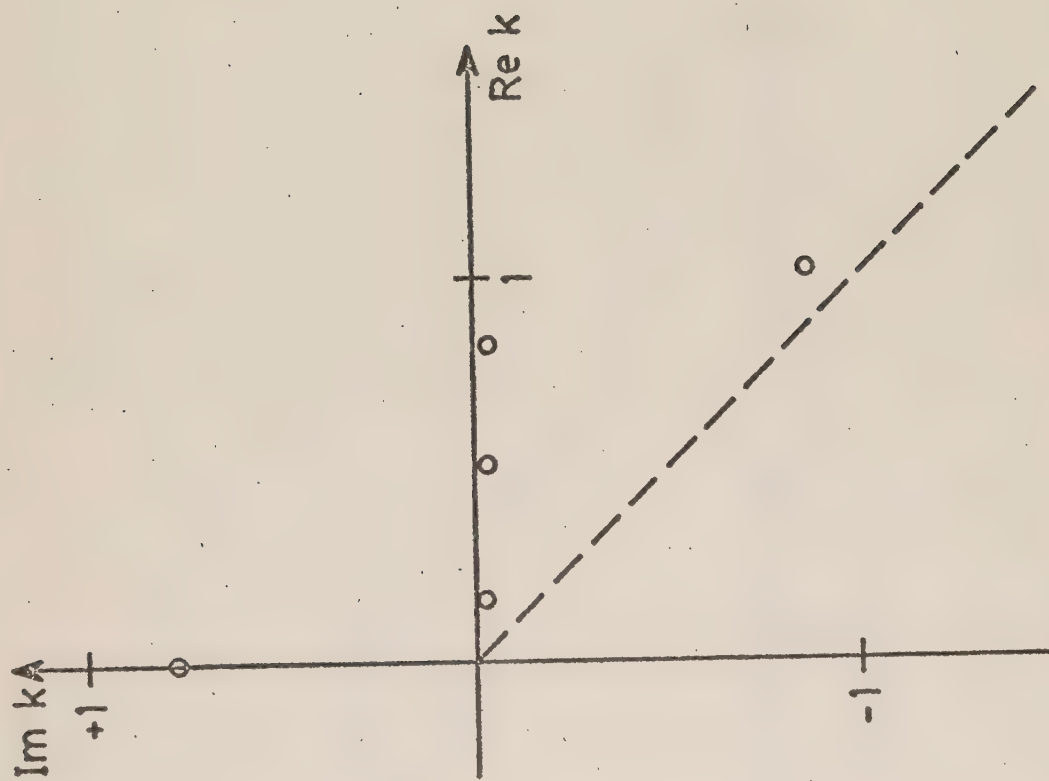


FIG.4

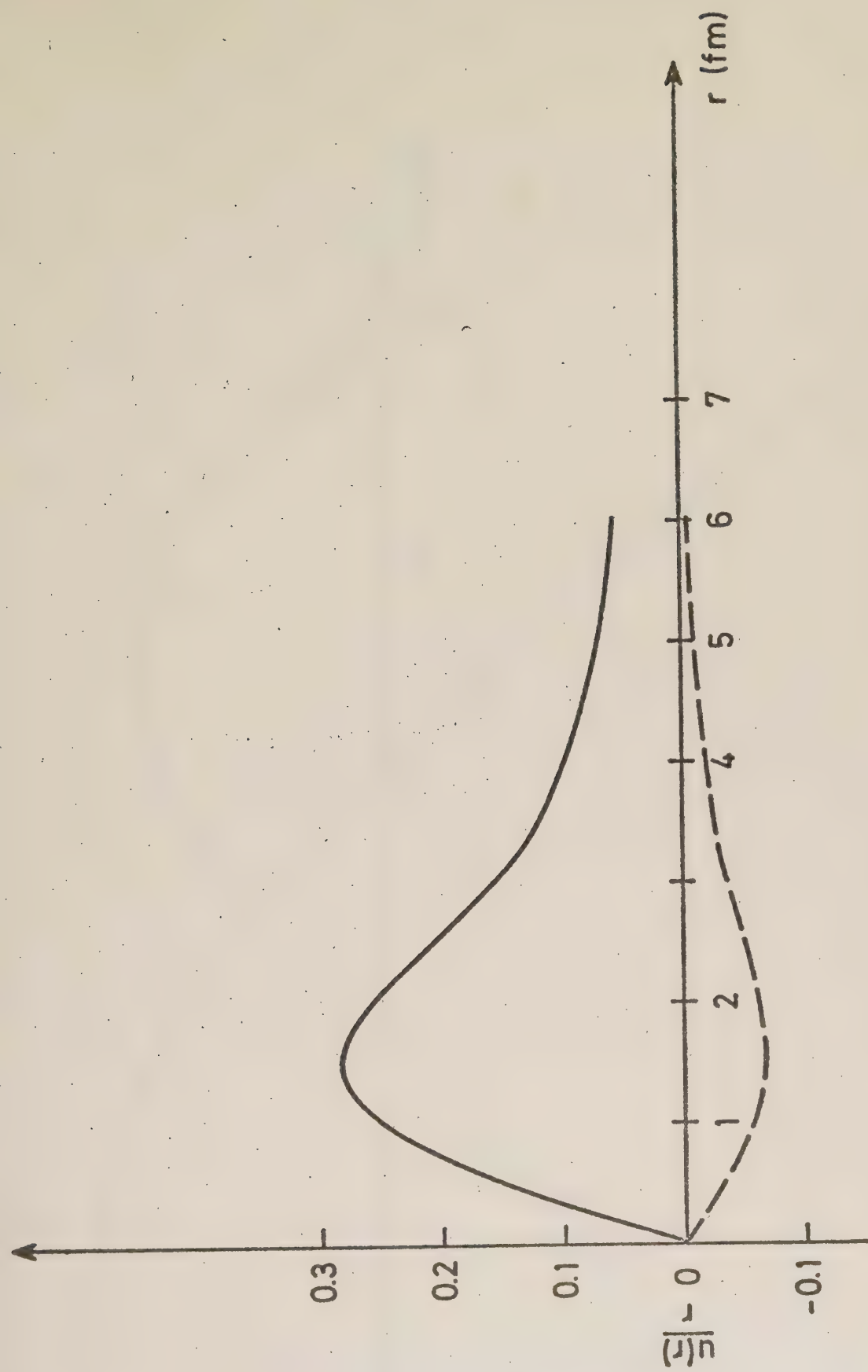


FIG. 5A

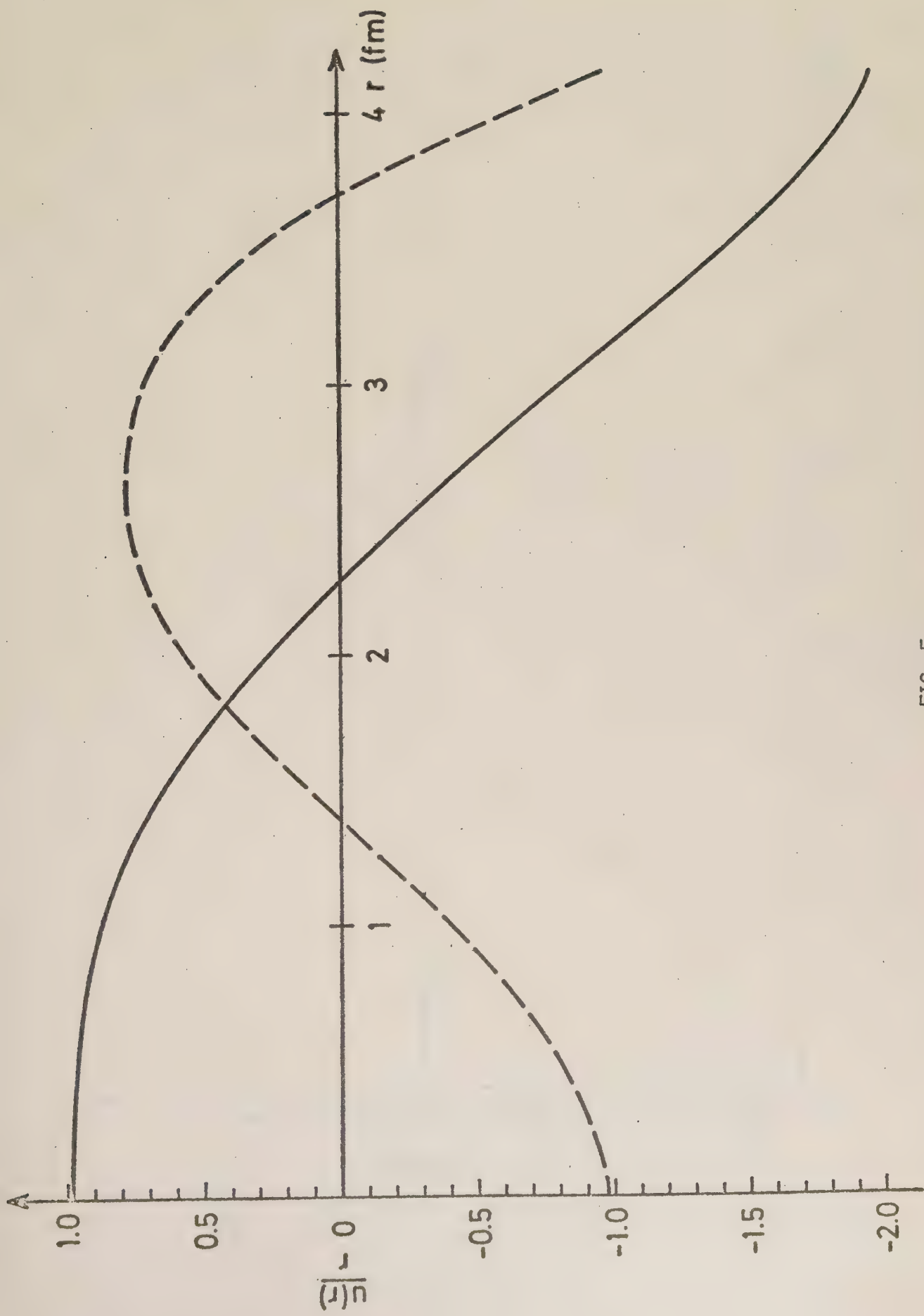


FIG. 5B

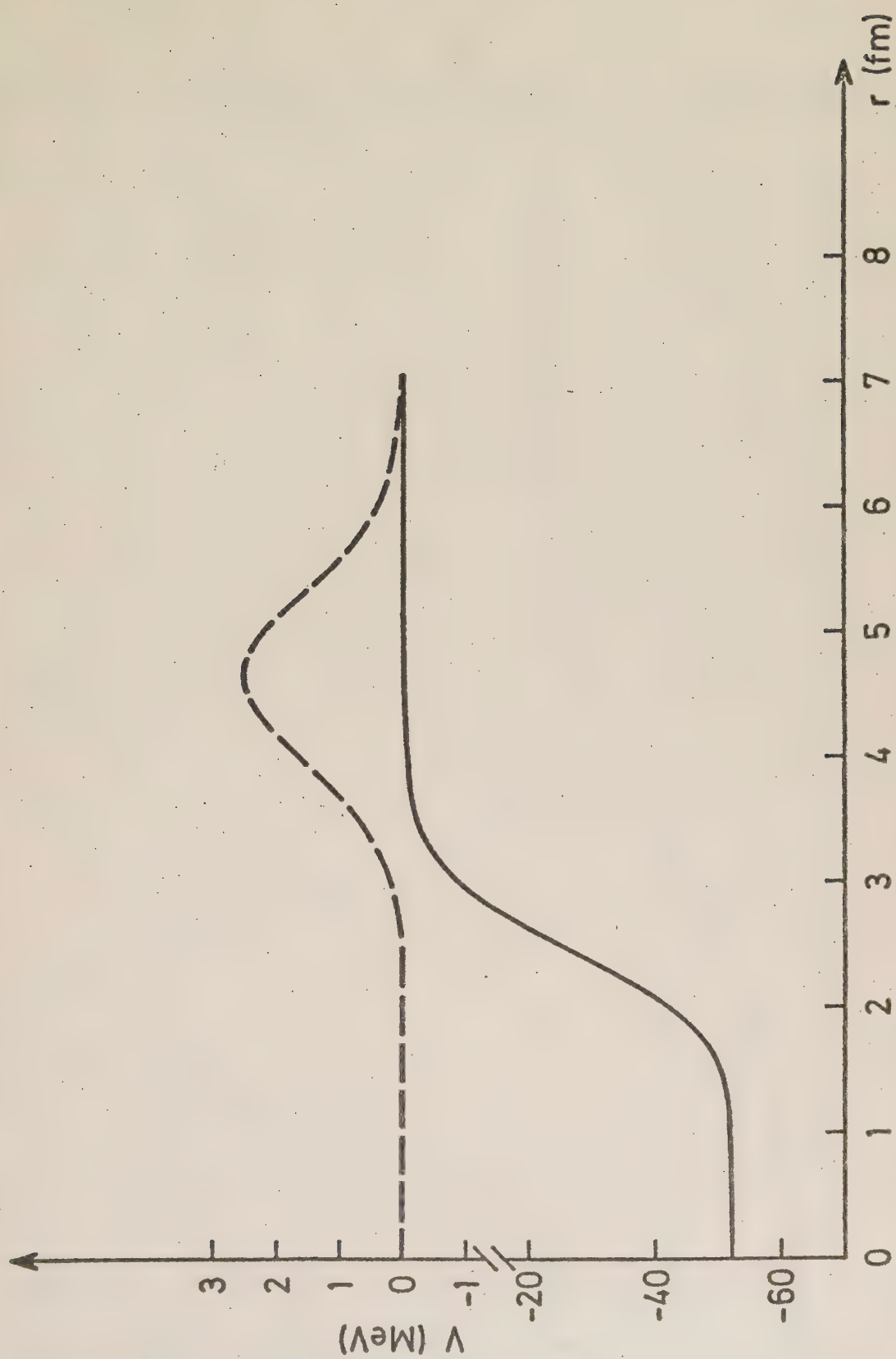


FIG. 6A

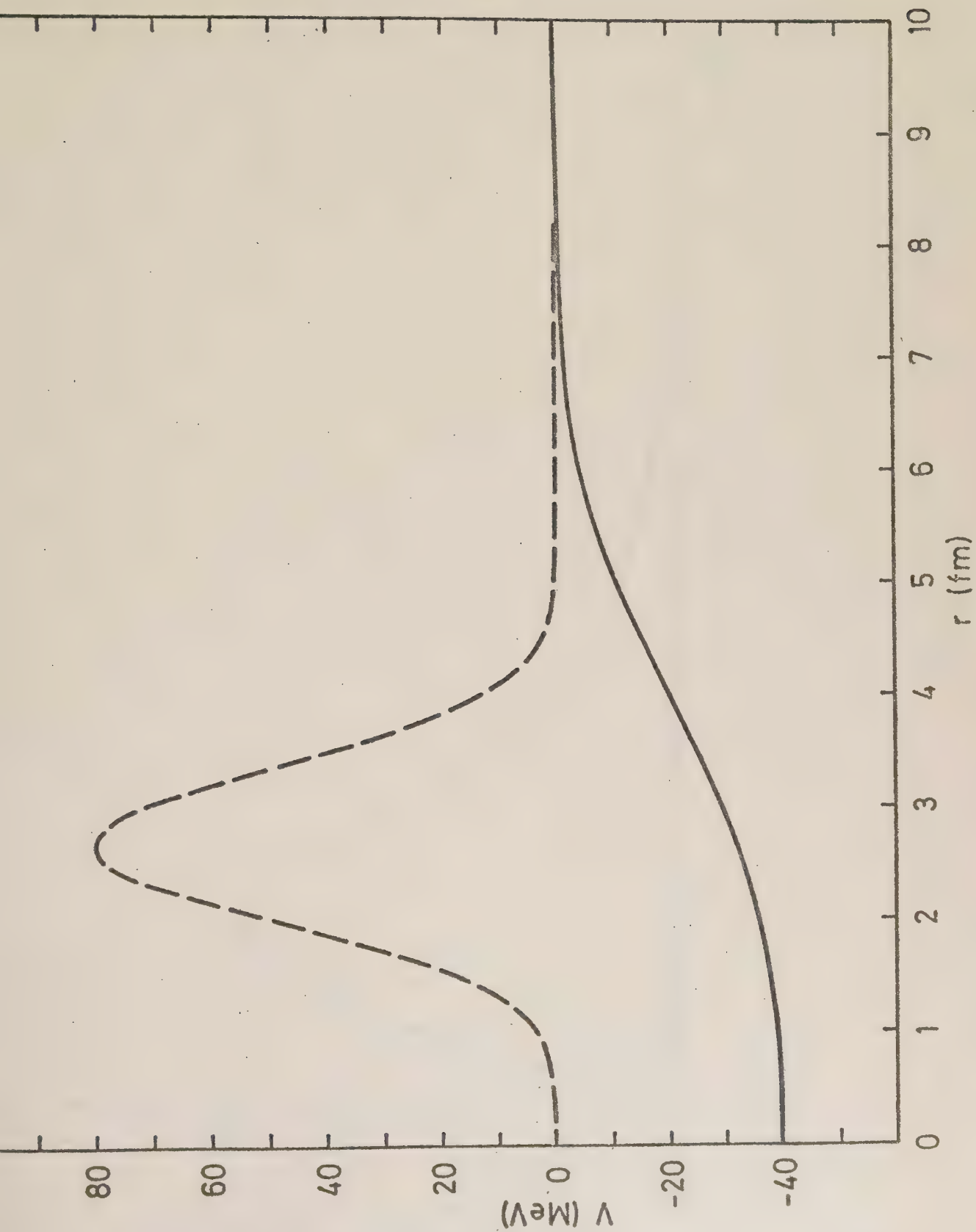


FIG. 6B

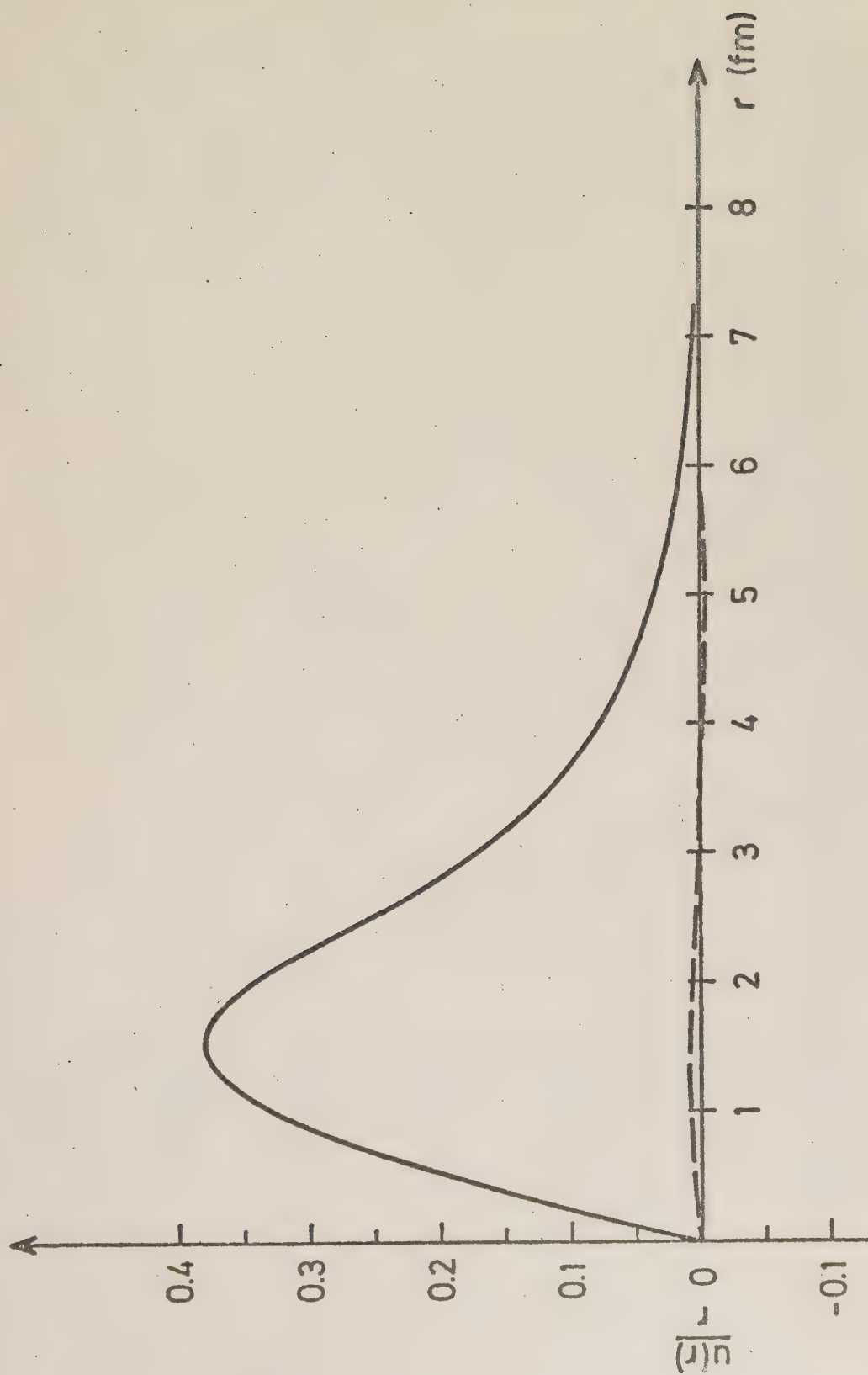


FIG. 7A

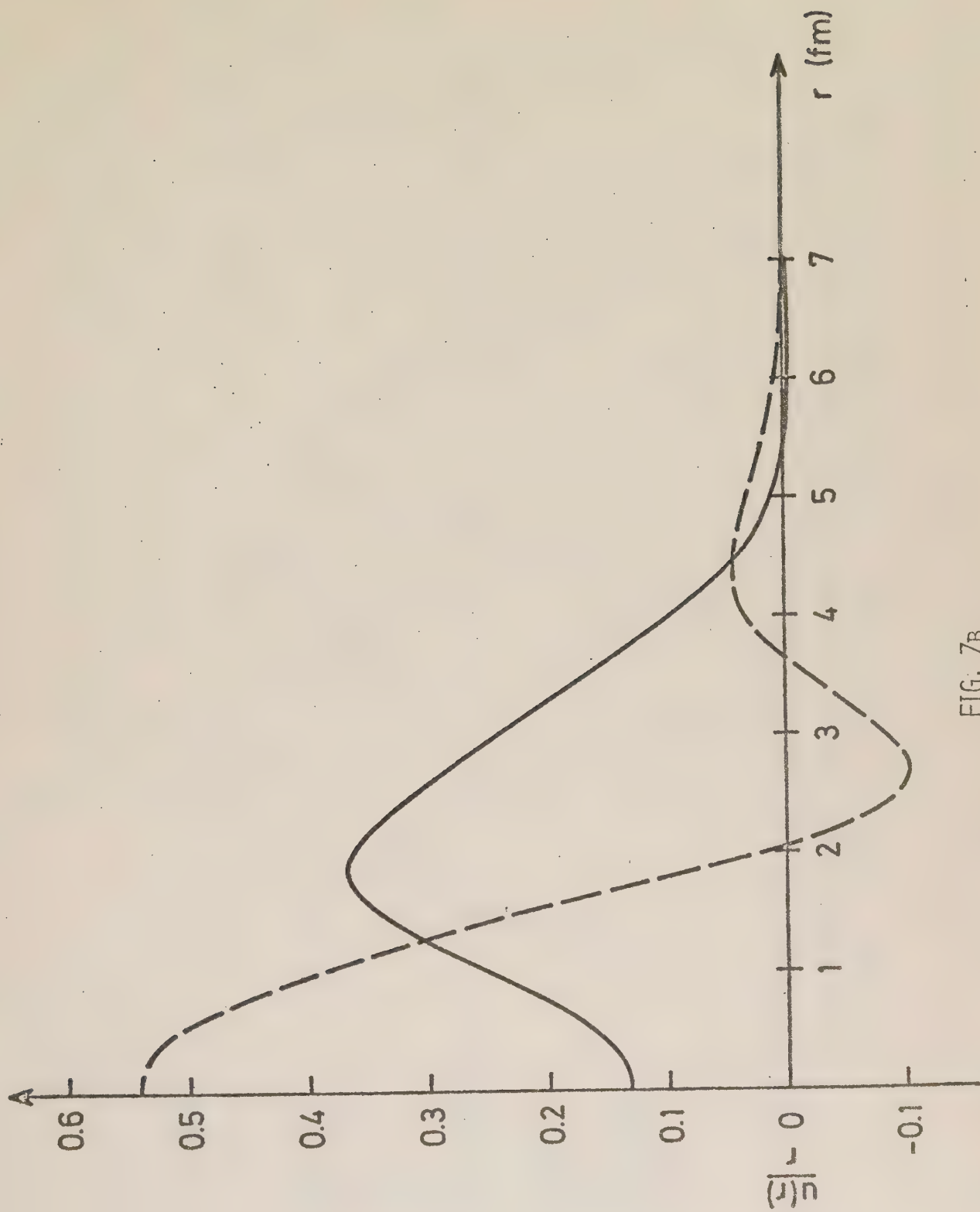


FIG. 7B

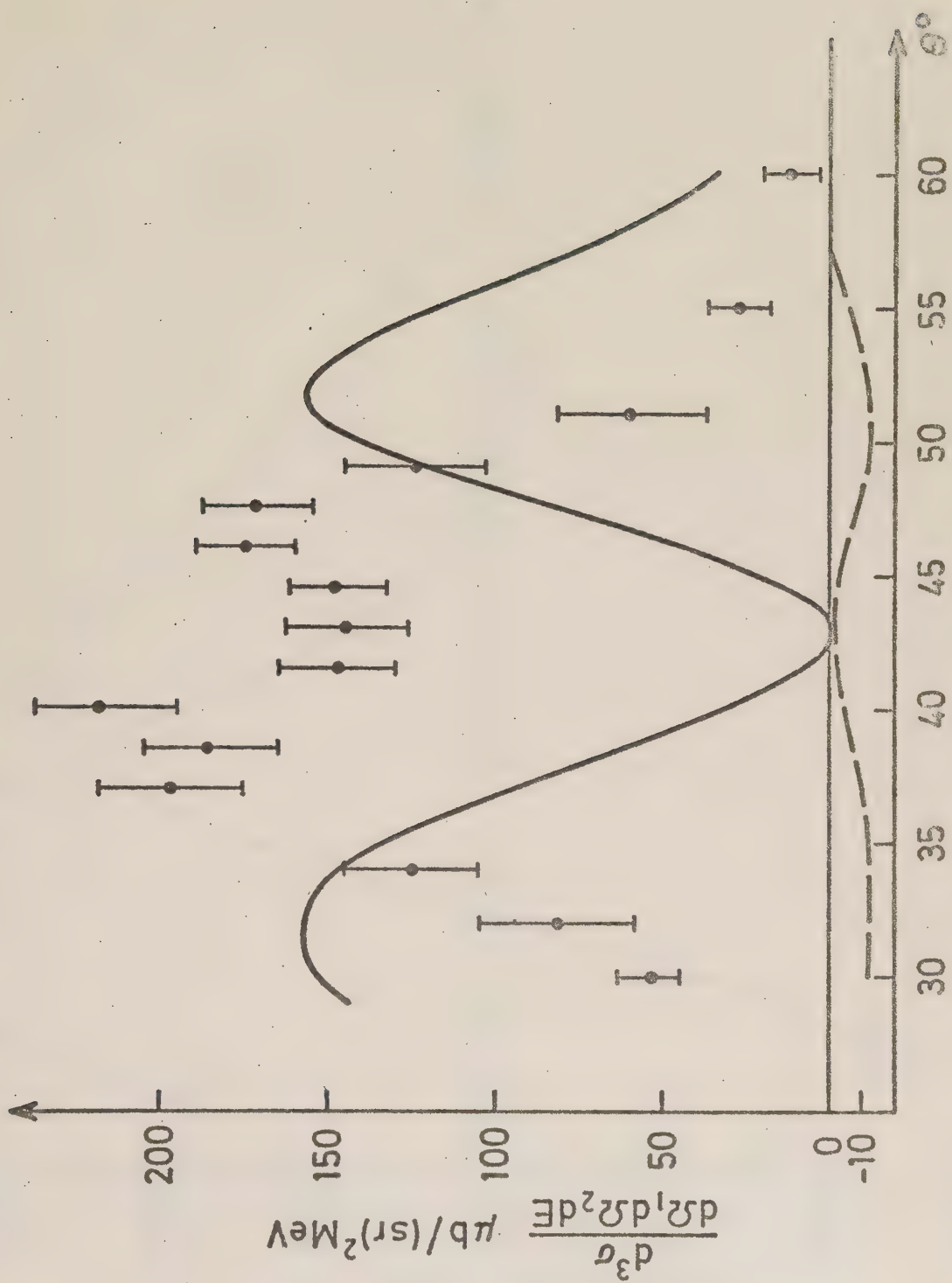
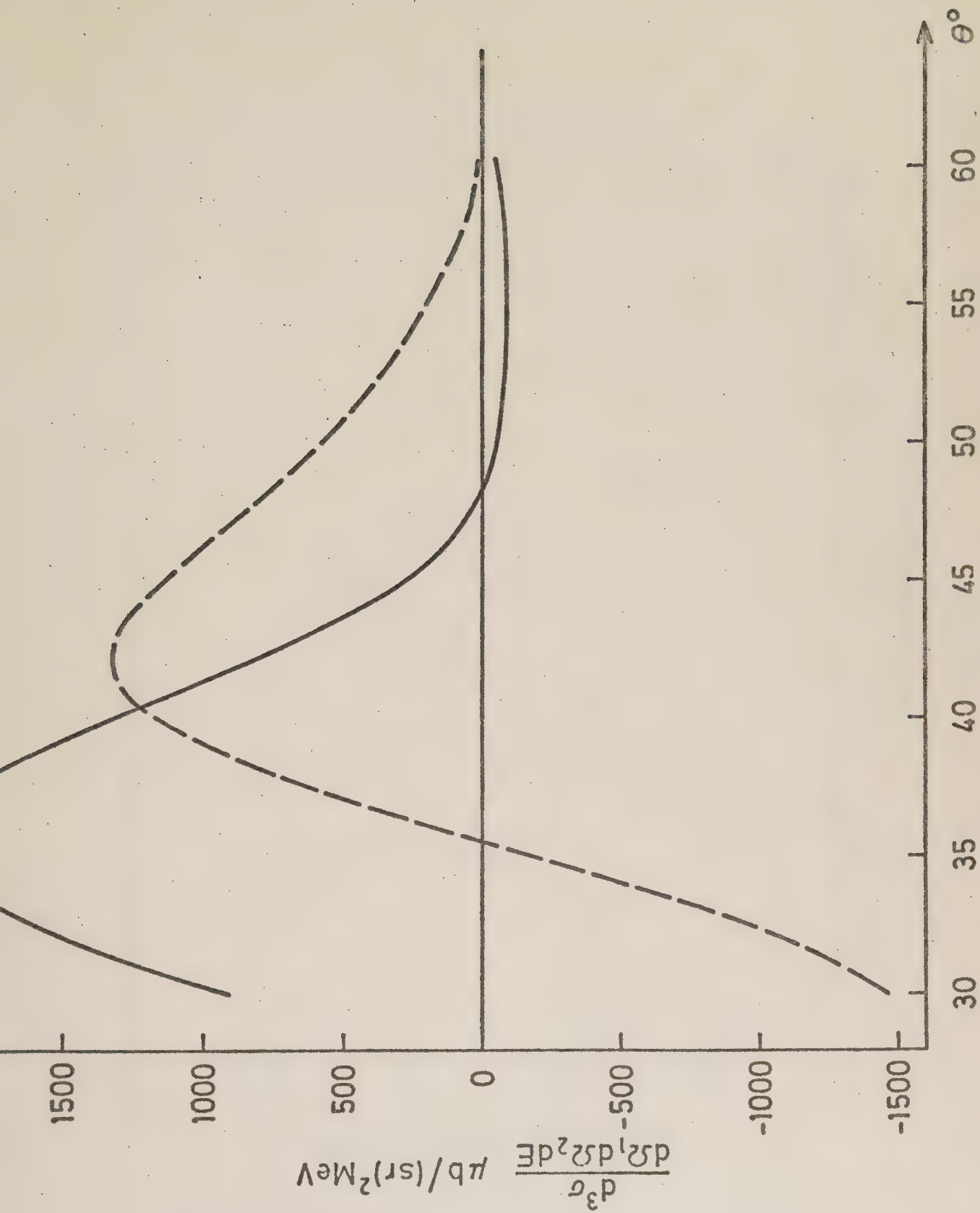


FIG. 8a



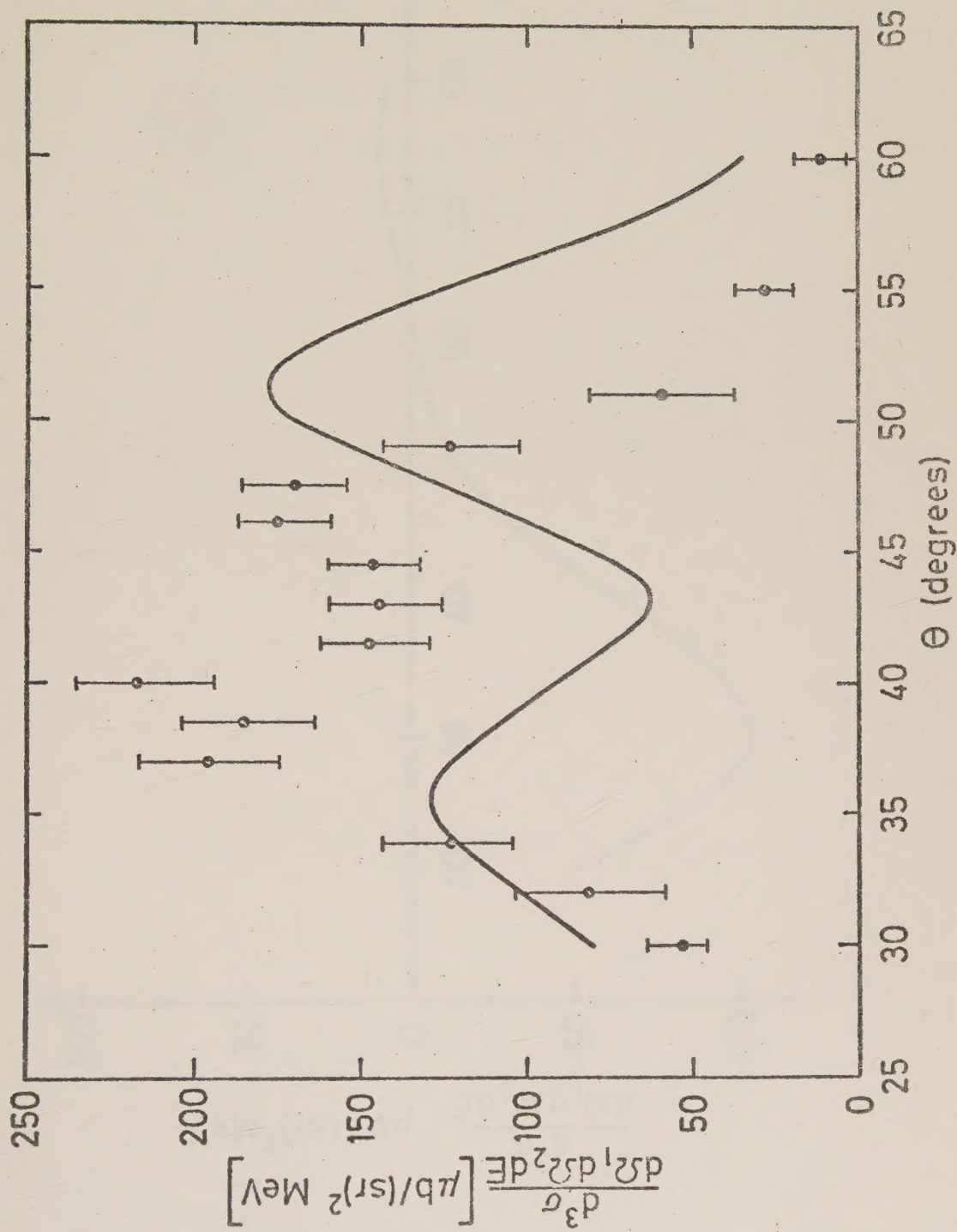


FIG. 9A

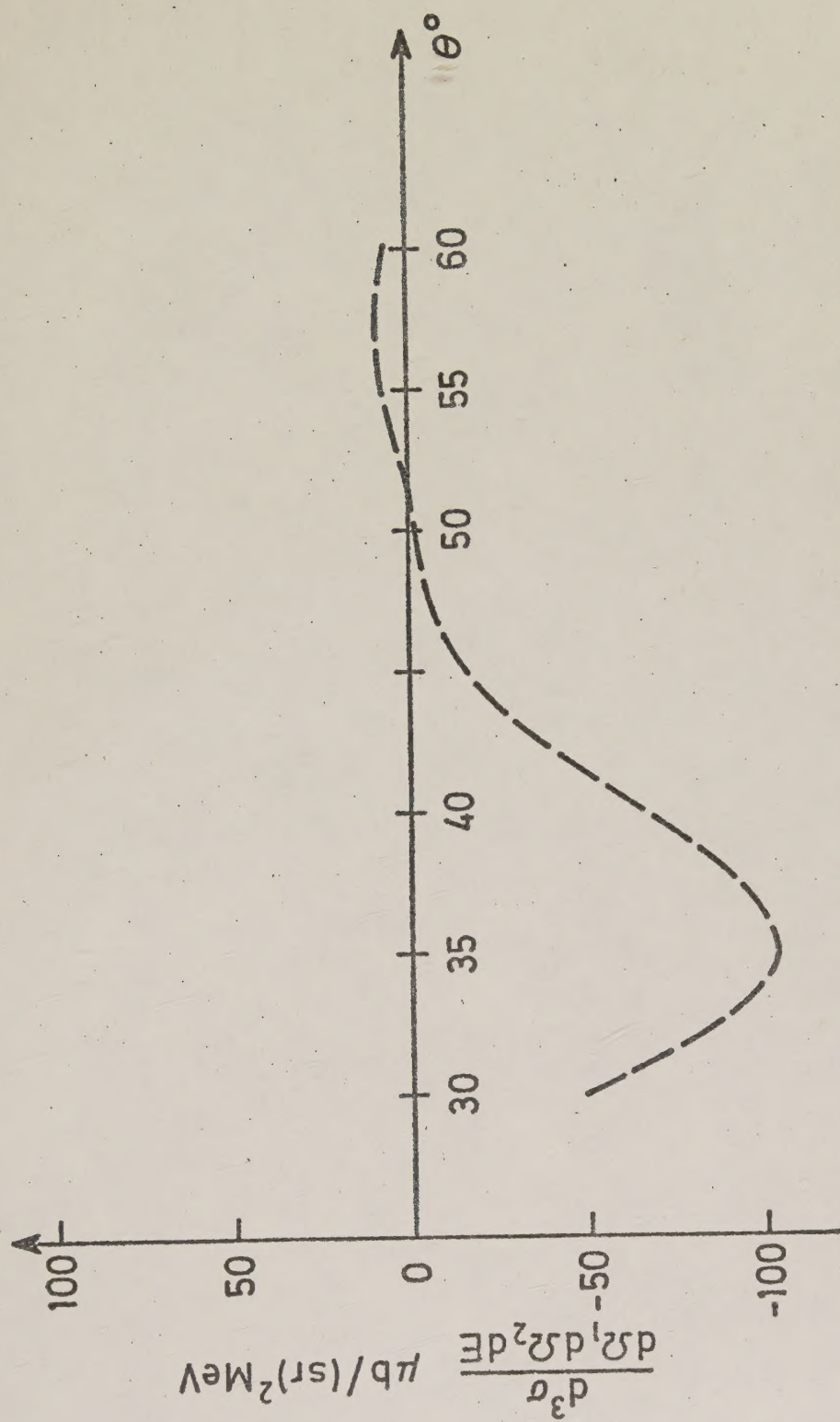


FIG. 9B

